Appendix G
OSS Average Response Interval Calculations and Graphics

It is of note that of the fifteen differences calculated, only two displayed negative differences, signaling even the possibility of any potential discrimination against the CLECs.

Time Series Analysis

Concerned with the possibility of a time dependence within the data, we employed time series analysis methodology. Figure 1 illustrates the average response interval differences for the four systems with "like-to-like" data. Figure 2 displays the average response interval differences for the overall series as a whole and also broken down by month.

A brief look at the graphs and the individual differences for each of the five series pointed out that the vast majority of days displayed positive differences. In fact, with only one exception, each day that exhibited a negative average response interval difference was always followed by a day with a positive difference. It was hard to judge from a preliminary study of the data and graphs if a time component was present, so we decided to engage in a more serious time series analysis.

The existence of unequal sample sizes for each day led us to reject the assumption that constant standard error between days existed and thus we had to conclude that the differences are not identically distributed. If we could estimate the daily variances, s_{1i}^2 and s_{2i}^2, we would correct this problem by standardizing each difference by dividing by an estimate of the standard error as in (1).

$$\frac{d_i}{\sqrt{s_p^2\left(\frac{1}{n_{1i}}+\frac{1}{n_{2i}}\right)}} \qquad (1)$$

Here s_p^2 is the pooled variance estimate, n_{1i} is the total number of BellSouth calls for the i^{th} date and n_{2i} is the total number of CLEC calls for the i^{th} date. Lacking this, we did the next best thing. We assumed that the variance for each response every day was constant, but unknown. Dividing each difference, d_i, by

$$\sqrt{\frac{1}{n_{1i}}+\frac{1}{n_{2i}}}$$

provides a rescaling that is proportional to the typical standardized value.

After rescaling the data, we dealt with the issue of missing observations. For a few dates within our time frame of interest, the CLECs data were present while BellSouth data were not. To correct this problem, we imputed on those days the mean values from the series. Using this method, we have a tendency to underestimate the standard error. An alternative may be to employ the EM algorithm to impute these values. However, we did not use the EM algorithm, because we felt our method was more conservative.

The autocorrelation and partial autocorrelation functions for each series were plotted using Interactive Time Series Modeling 6.0 (ITSM) software in an attempt to identify the existence of a time dependent process. Table 2 illustrates the results of our time series analysis and the associated parameters.

Table 4 - Test Results

Overall

Month	Test Statistic	df	P-value (percent)
July	0.5396	22	29.7446
August	3.7770	20	0.0592
September	1.2031	21	12.1163

ATLAS

Month	Test Statistic	df	P-value (percent)
July	3.2101	22	0.2017
August	3.2453	20	0.2027
September	3.0683	21	0.2917

DSAP

Month	Test Statistic	df	P-value (percent)
July	3.0418	22	0.2992
August	4.2157	20	0.0212
September	1.9928	21	2.9717

RSAG(By ADDR)

Month	Test Statistic	df	P-value (percent)
July	4.0417	22	0.0272
August	6.5352	20	0.0001
September	5.6244	21	0.0007

RSAG(By TN)

Month	Test Statistic	df	P-value (percent)
July	-0.8686	22	19.7226
August	1.0576	20	15.1419
September	-0.6530	21	26.0422

Of the fifteen test statistics calculated, only two had negative test values and these were quite small. Furthermore, the P-values for the two negative tests were quite large indicating that there was not enough evidence to suggest any significant differences.

References:

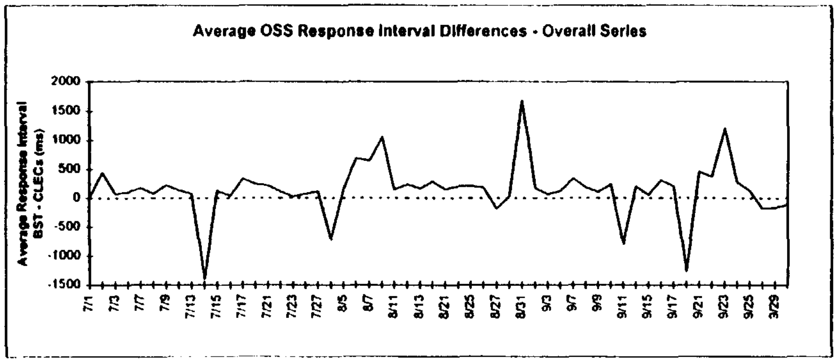
Brockwell, Peter J. and Davis, Richard A., *A First Course in Time Series Analysis*, Springer-Verlag New York, Inc., New York, 1995.

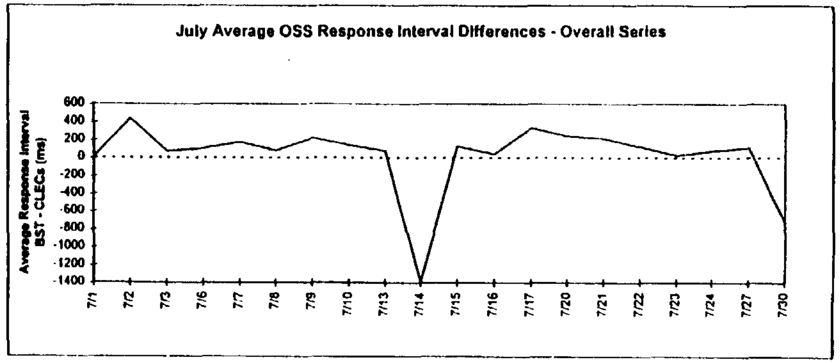
Wei, William S., *Time Series Analysis - Univariate and Multivariate Methods*, Addison-Wesley Publishing Company, Inc., Redwood City, California, 1990.

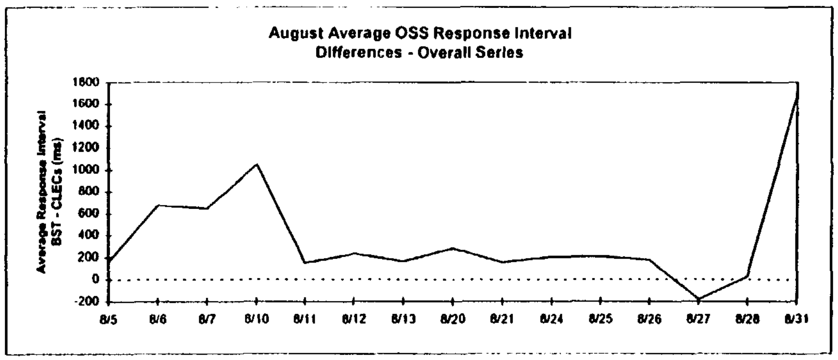
BellSouth Local Competition Operational Readiness - Prepared for the United States Department of Justice, 1997

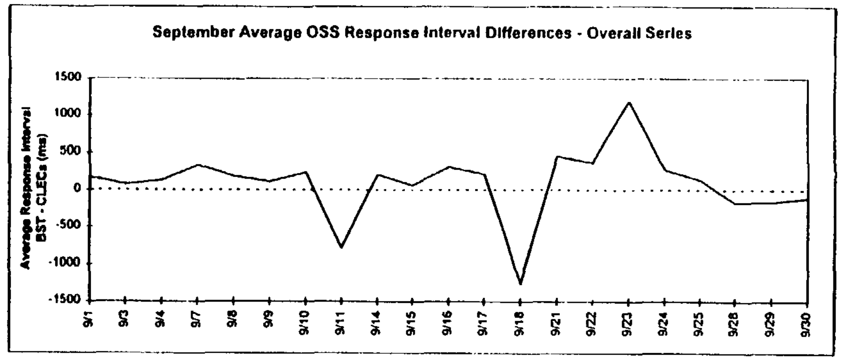
SAS Institute Inc., *SAS/ETS® User's Guide, Version 6, Second Edition*, Cary, NC: SAS Institute Inc., 1993.

Figure 2 - Overall Time Series of Average OSS Differences - BST minus CLECs





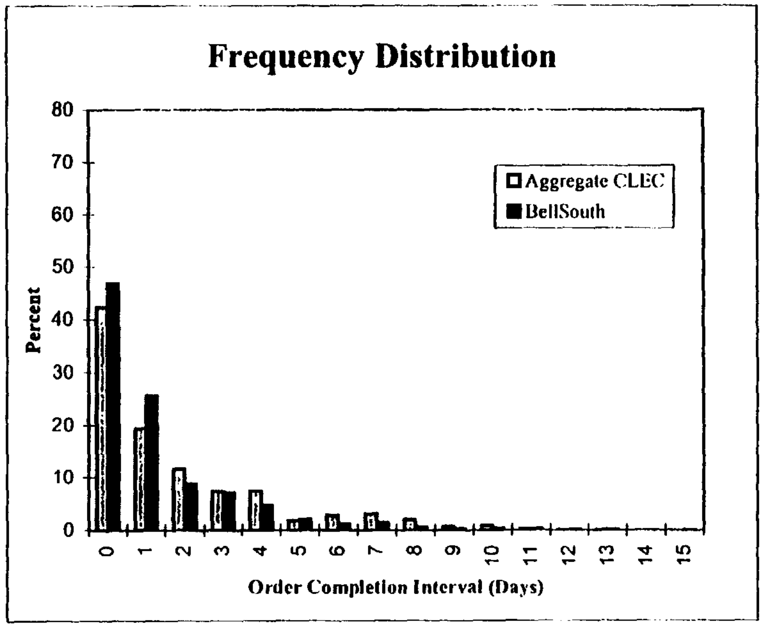


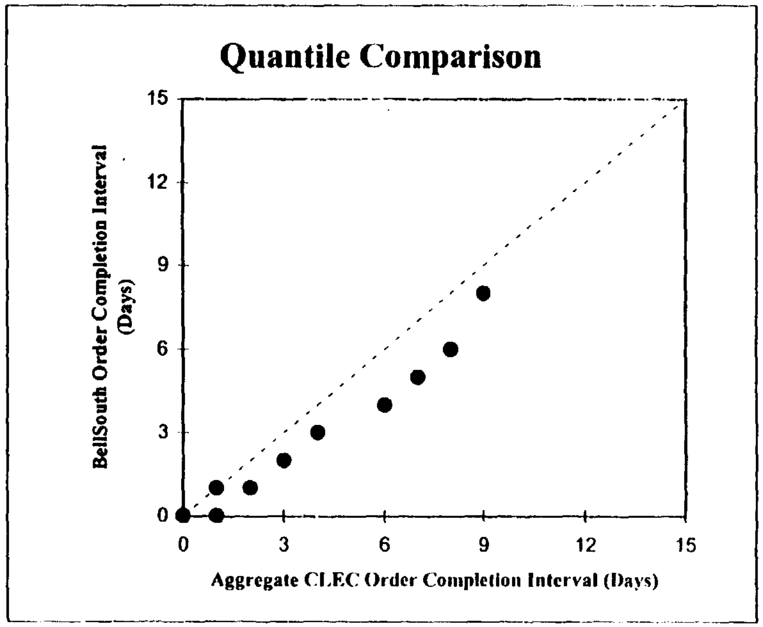


Appendix H
LATA - August Graphics

I. Graphical Representations

Adjusted
August BellSouth and CLEC Completion Interval-Provisioning
Shreveport Cases





Descriptive Measures

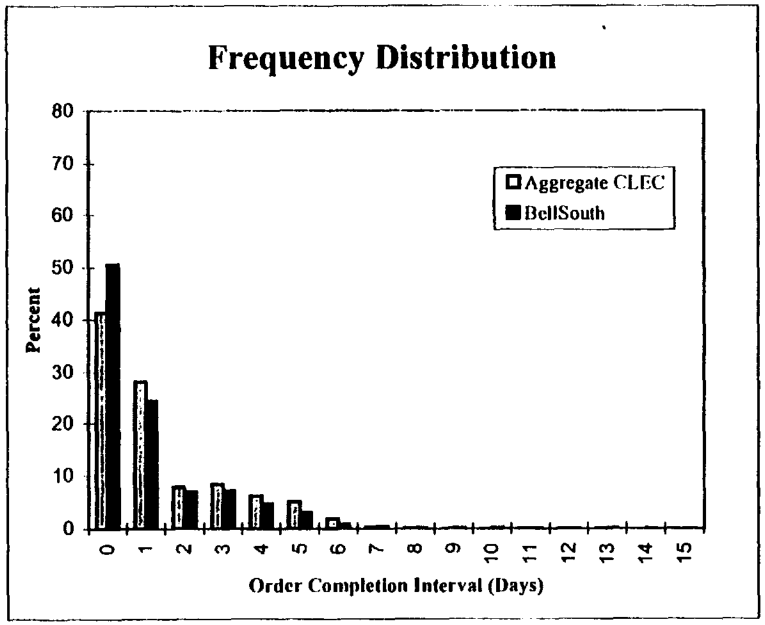
Service Provider	Mean	Standard Deviation
BST	1.41	2.54
CLEC	1.82	2.54
Difference	-0.42	

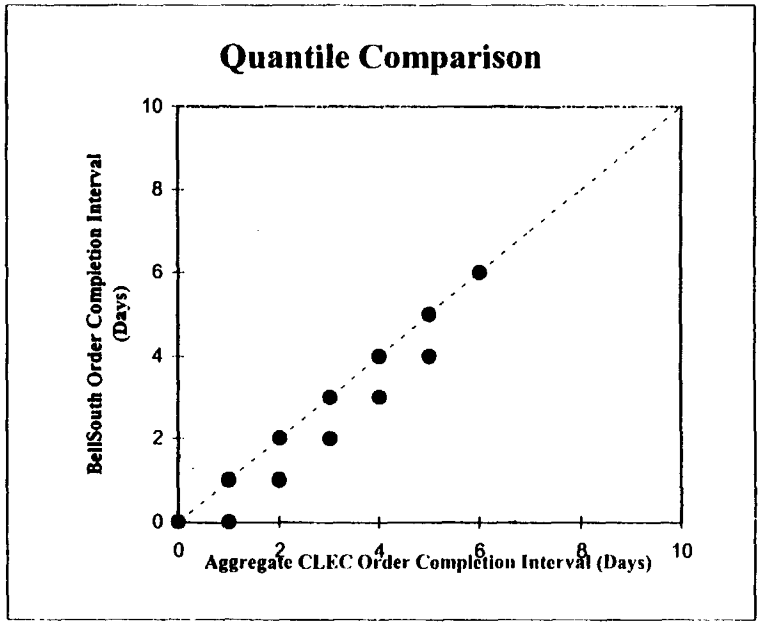
Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	-11.44	0.0000
FCC	-11.44	0.0000
BST	-4.54	0.0046

Data used in analysis does not include any records with missed appointments due to customer rescheduling or records corresponding to official services.
The application of statistical trimming removed records with completion interval-provisioning of above 99 days. This resulted in the removal of no CLEC records and 0.004% of the BellSouth records.

Adjusted
August BellSouth and CLEC Completion Interval-Provisioning
Lafayette Cases





Descriptive Measures

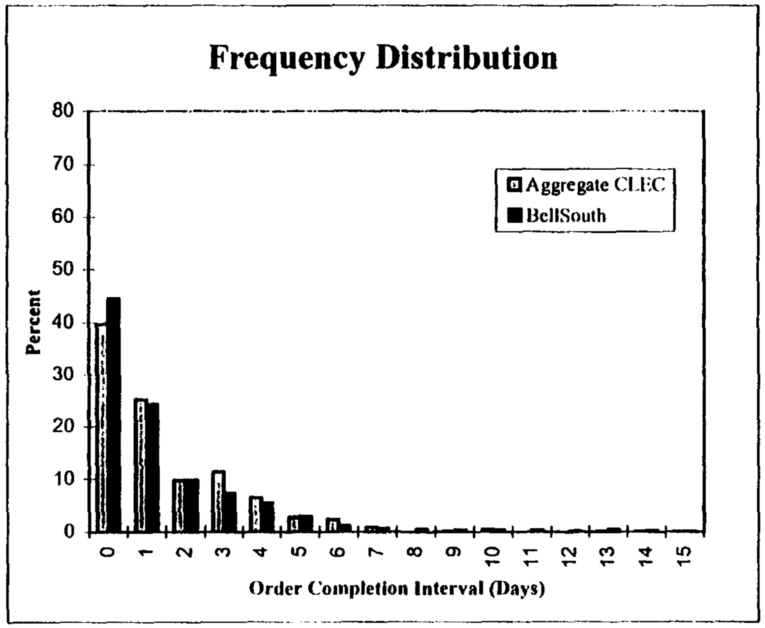
Service Provider	Mean	Standard Deviation
BST	1.21	2.24
CLEC	1.38	1.71
Difference	-0.17	

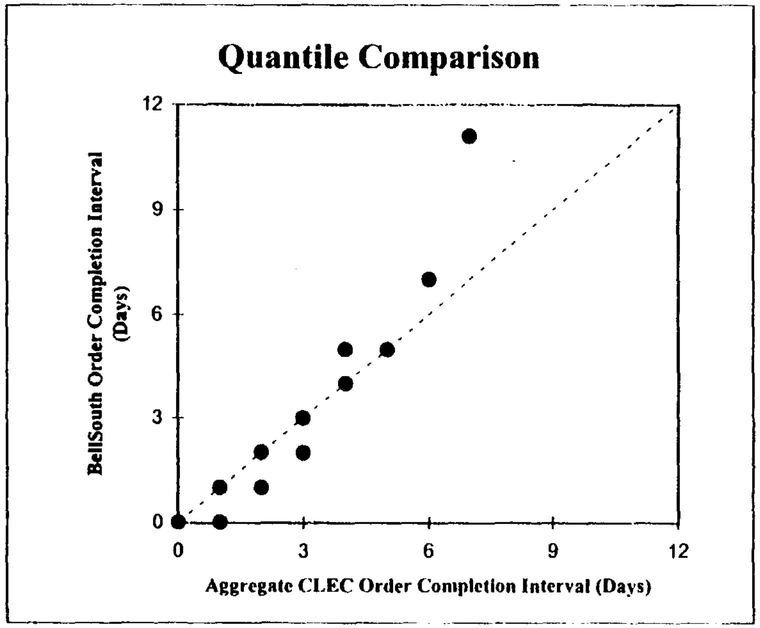
Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	-3.99	0.0033
FCC	-4.03	0.0028
BST	-1.62	5.7944

Data used in analysis does not include any records with missed appointments due to customer rescheduling or records corresponding to official services.
The application of statistical trimming removed records with completion interval-provisioning of above 99 days. This resulted in the removal of no CLEC records and 0.004% of the BellSouth records.

Adjusted
August BellSouth and CLEC Completion Interval-Provisioning
New Orleans Cases





Descriptive Measures

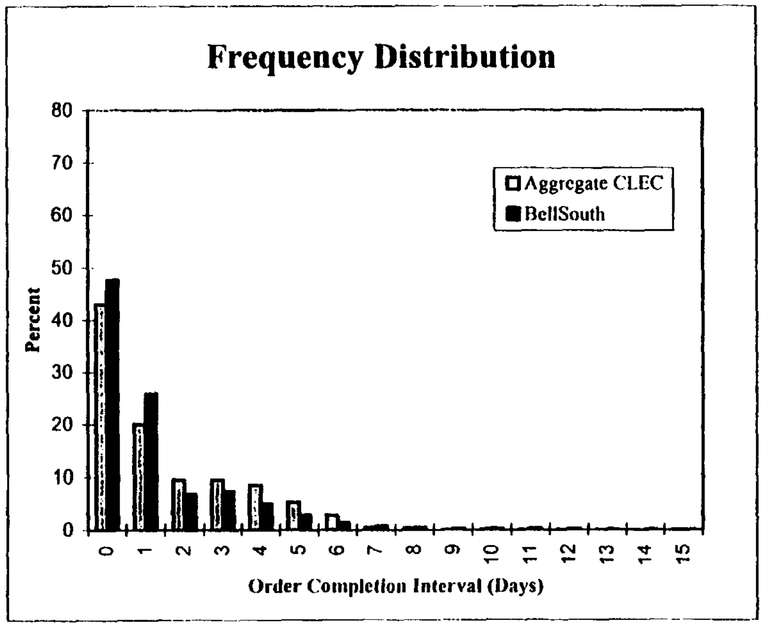
Service Provider	Mean	Standard Deviation
BST	1.70	3.53
CLEC	1.57	2.25
Difference	0.12	

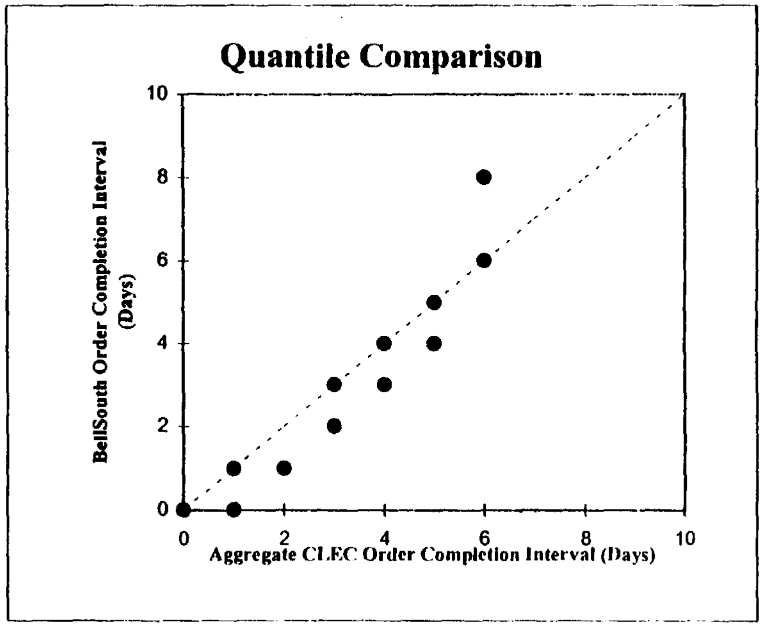
Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	2.55	0.5418
FCC	2.57	0.5065
BST	1.93	3.1819

Data used in analysis does not include any records with missed appointments due to customer rescheduling or records corresponding to official services.
The application of statistical trimming removed records with completion interval-provisioning of above 99 days. This resulted in the removal of no CLEC records and 0.004% of the BellSouth records.

Adjusted
August BellSouth and CLEC Completion Interval-Provisioning
Baton Rouge Cases





Descriptive Measures

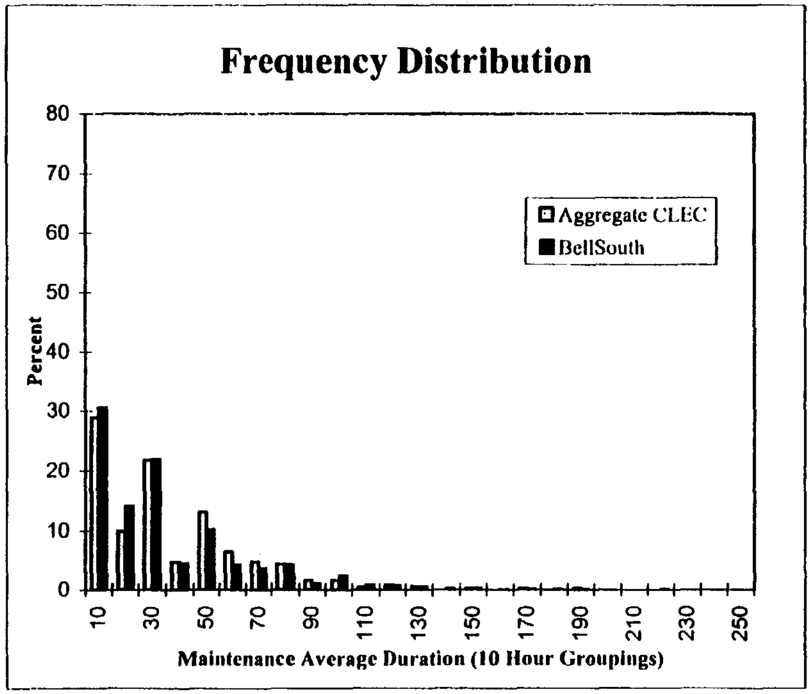
Service Provider	Mean	Standard Deviation
BST	1.44	3.00
CLEC	1.58	2.19
Difference	-0.14	

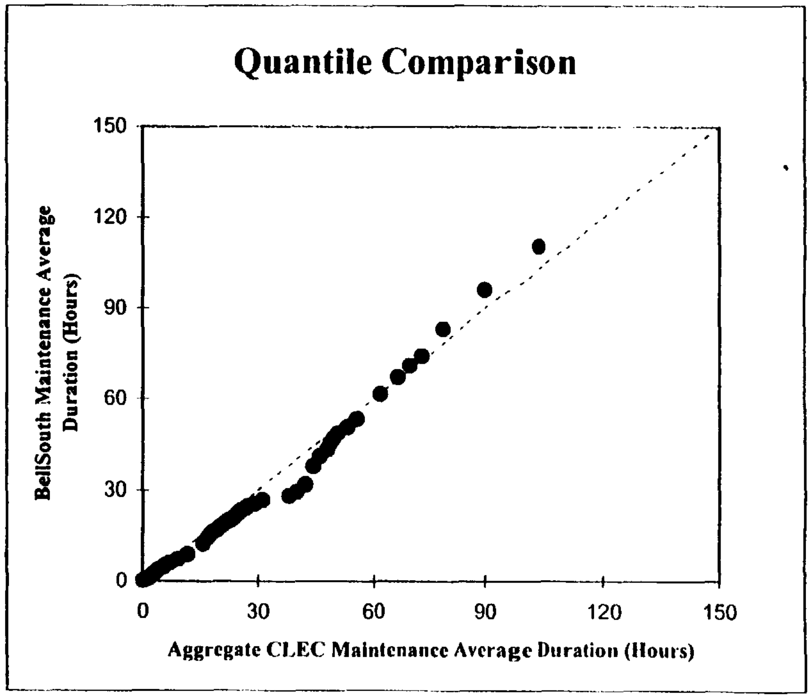
Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	-2.33	0.9806
FCC	-2.35	0.9268
BST	-0.78	22.0778

Data used in analysis does not include any records with missed appointments due to customer rescheduling or records corresponding to official services.
The application of statistical trimming removed records with completion interval-provisioning of above 99 days. This resulted in the removal of no CLEC records and 0.004% of the BellSouth records.

Adjusted
August BellSouth and CLEC Average Duration-Maintenance
Shreveport





Descriptive Measures

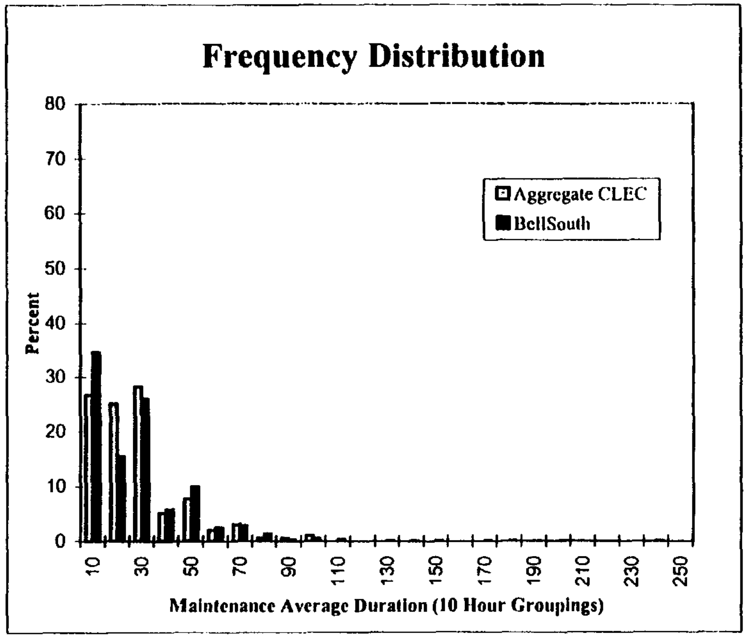
Service Provider	Mean	Standard Deviation
BST	29.48	29.34
CLEC	31.48	28.47
Difference	-2.00	

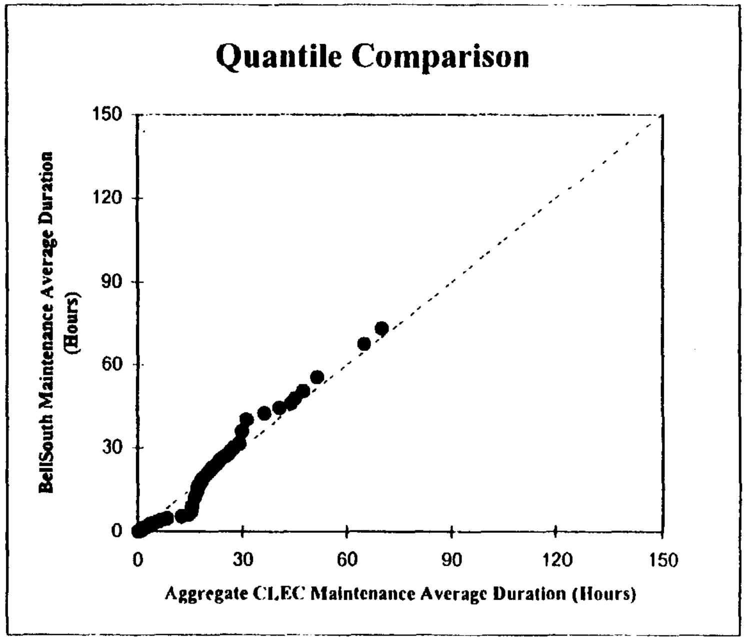
Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	-1.53	6.3200
FCC	-1.53	6.3058
BST	-1.20	12.0398

Data used in analysis includes only direct customer reports. The results exclude in public service lines and durations > 240 hours

Adjusted
August BellSouth and CLEC Average Duration-Maintenance
Lafayette





Descriptive Measures

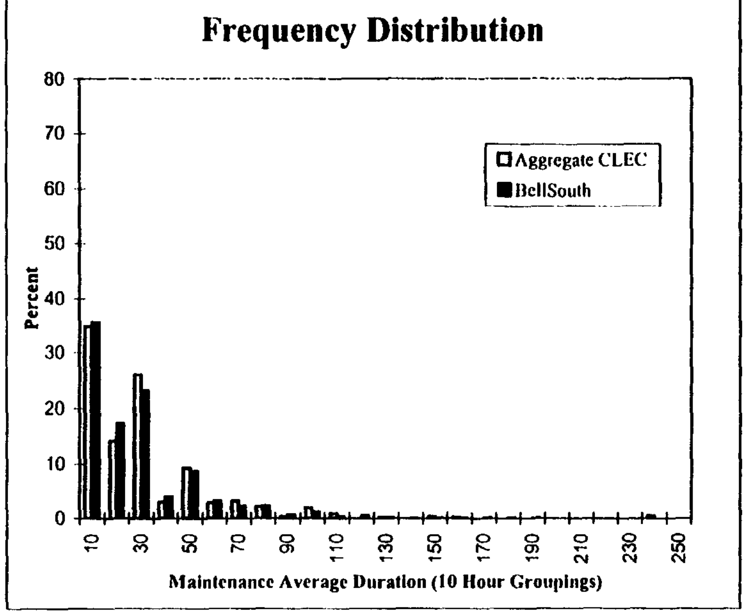
Service Provider	Mean	Standard Deviation
BST	22.21	21.24
CLEC	21.93	17.99
Difference	0.28	

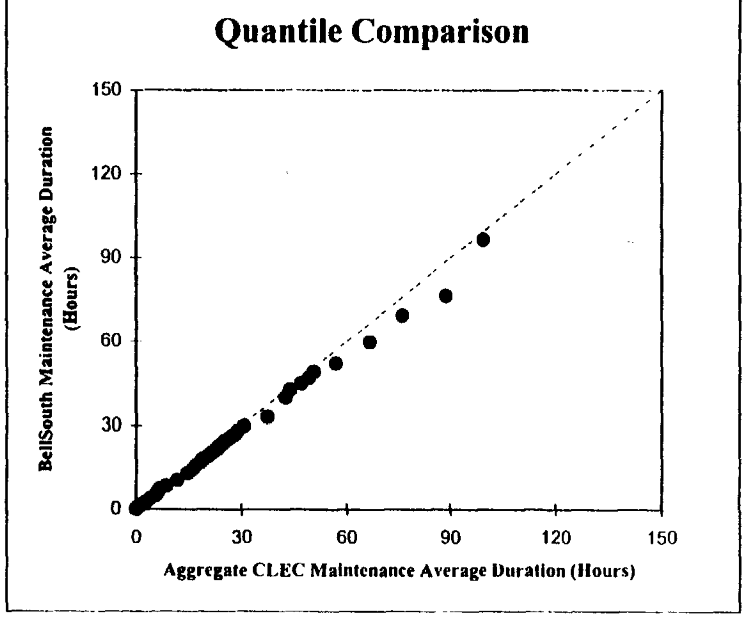
Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	0.18	42.7508
FCC	0.18	42.7358
BST	0.16	43.8402

Data used in analysis includes only direct customer reports. The results exclude in public service lines and durations > 240 hours

Adjusted
August BellSouth and CLEC Average Duration-Maintenance
New Orleans





Descriptive Measures

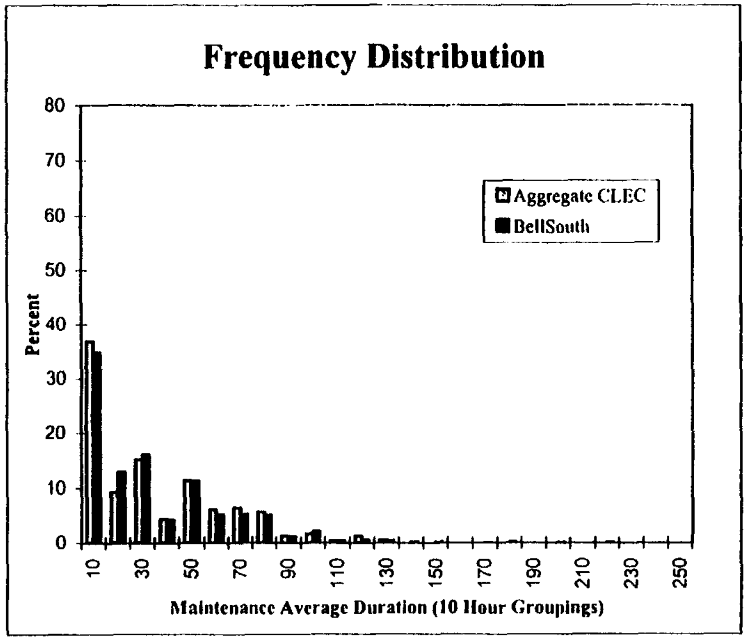
Service Provider	Mean	Standard Deviation
BST	23.58	25.06
CLEC	25.55	28.81
Difference	-1.97	

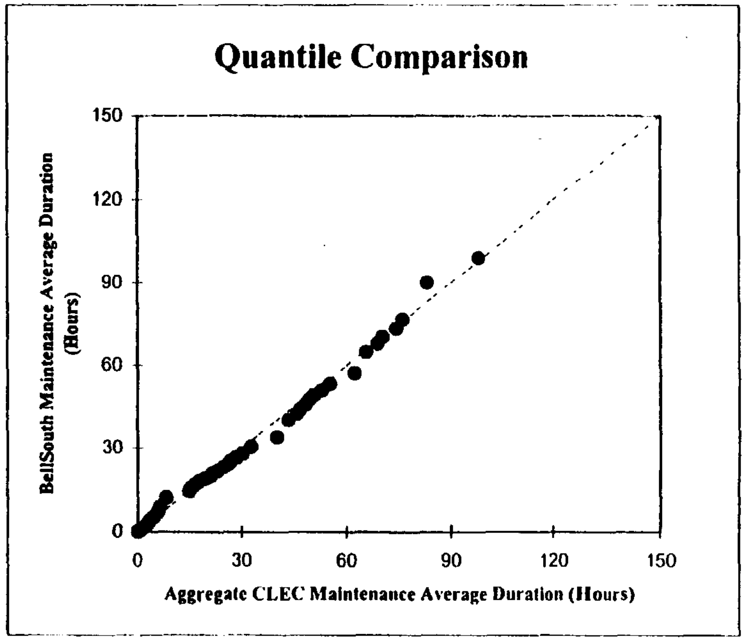
Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	-1.68	4.6442
FCC	-1.68	4.6897
BST	-1.57	6.4115

Data used in analysis includes only direct customer reports. The results exclude in public service lines and durations > 240 hours

Adjusted
August BellSouth and CLEC Average Duration-Maintenance
Baton Rouge





Descriptive Measures

Service Provider	Mean	Standard Deviation
BST	29.25	28.98
CLEC	29.76	28.20
Difference	-0.51	

Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	-0.27	39.2847
FCC	-0.27	39.2790
BST	-0.24	40.8240

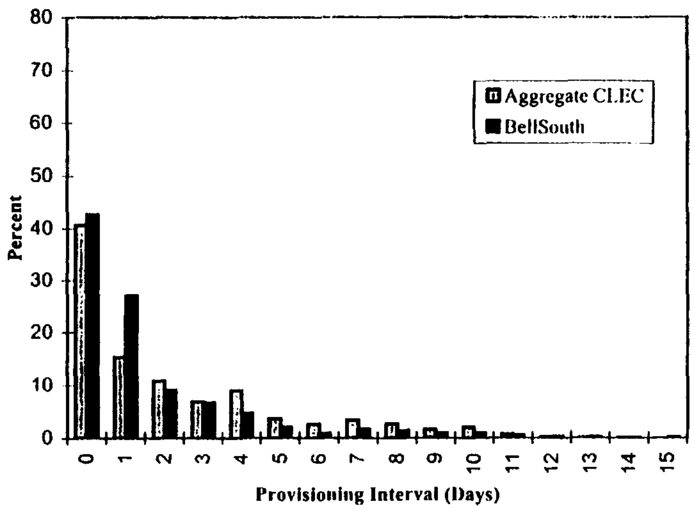
Data used in analysis includes only direct customer reports. The results exclude in public service lines and durations > 240 hours

Appendix I
LATA - September Graphics

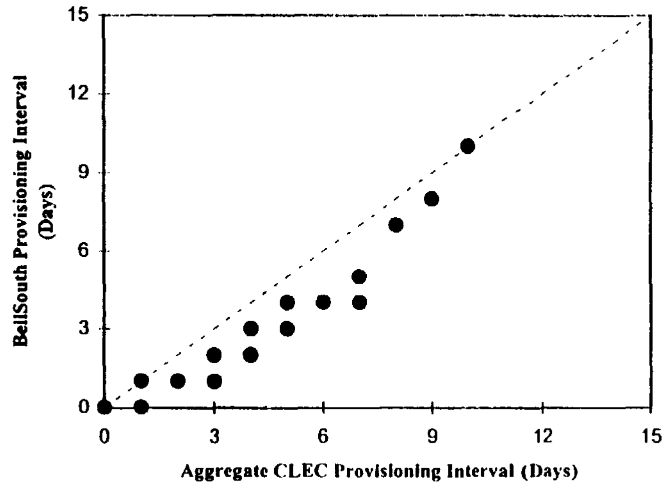
I. Graphical Representations

Adjusted
September BellSouth and CLEC Completion Interval-Provisioning
Shreveport Cases

Frequency Distribution



Quantile Comparison



Descriptive Measures

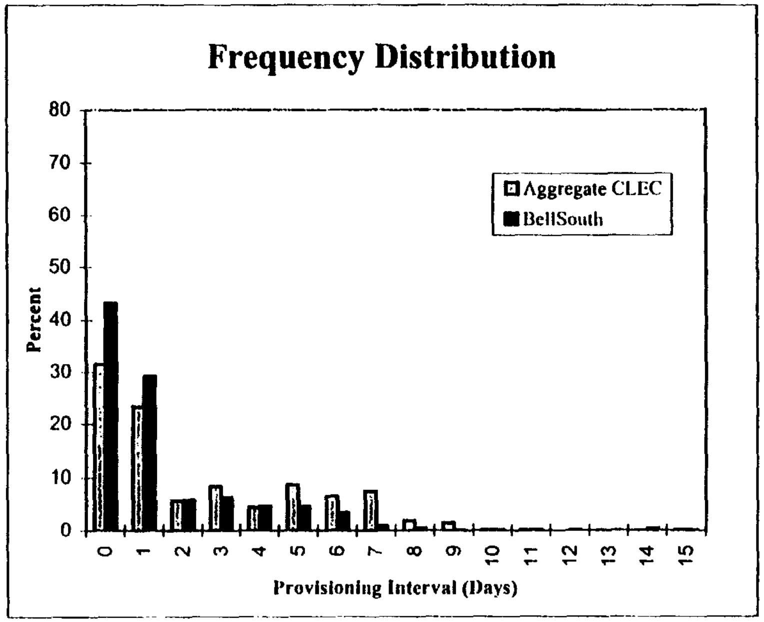
Service Provider	Mean	Standard Deviation
BST	1.70	3.00
CLEC	2.23	2.88
Difference	-0.53	

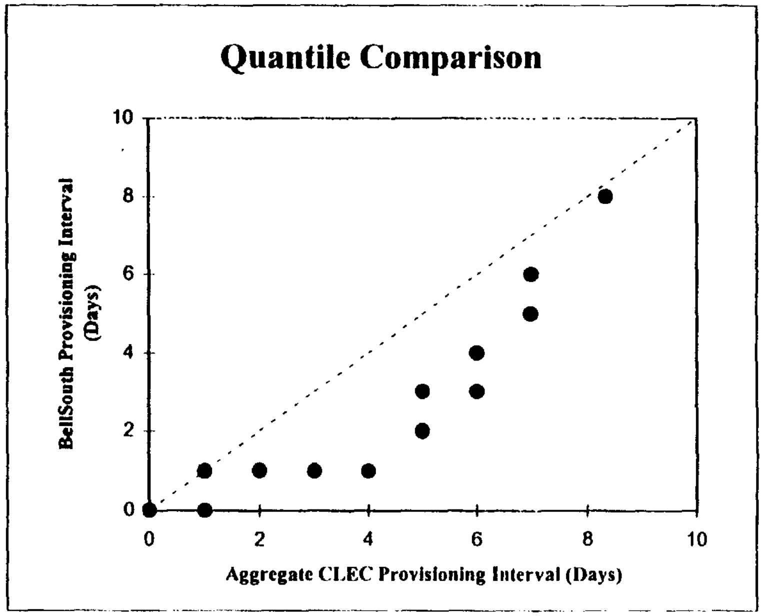
Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	-12.53	0.0000
FCC	-12.56	0.0000
BST	-4.18	0.0121

Data used in analysis does not include any records with missed appointments due to customer rescheduling or records corresponding to official services.
The application of statistical trimming removed records with completion interval-provisioning of above 99 days. This resulted in the removal of no CLEC records and 0.004% of the BellSouth records.

Adjusted
September BellSouth and CLEC Completion Interval-Provisioning
Lafayette Cases





Descriptive Measures

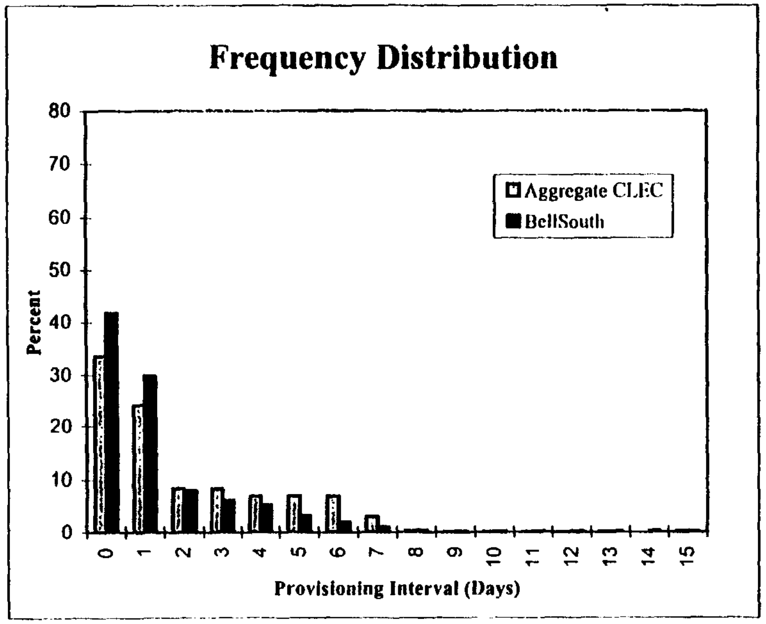
Service Provider	Mean	Standard Deviation
BST	1.56	2.59
CLEC	2.48	2.73
Difference	-0.93	

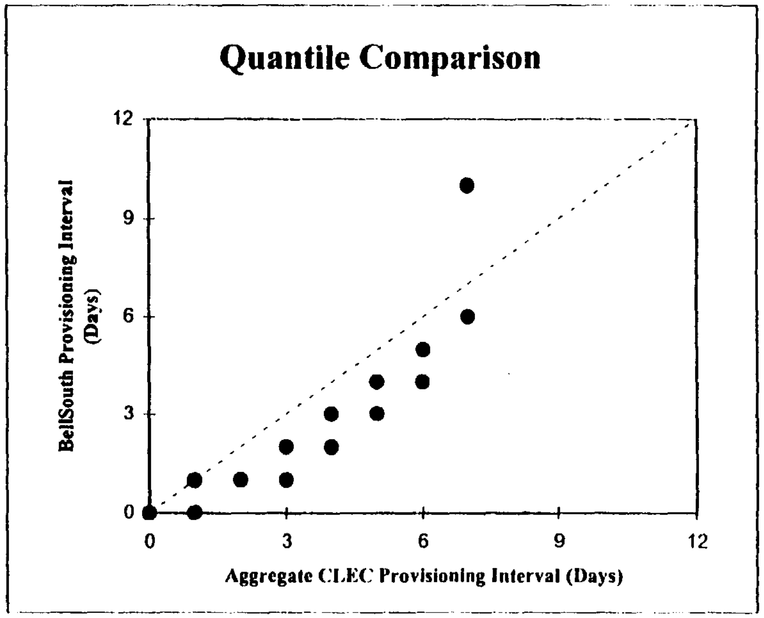
Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	-17.69	0.0000
FCC	-17.64	0.0000
BST	-4.69	0.0030

Data used in analysis does not include any records with missed appointments due to customer rescheduling or records corresponding to official services.
The application of statistical trimming removed records with completion interval-provisioning of above 99 days. This resulted in the removal of no CLEC records and 0.004% of the BellSouth records.

Adjusted
September BellSouth and CLEC Completion Interval-Provisioning
New Orleans Cases





Descriptive Measures

Service Provider	Mean	Standard Deviation
BST	1.64	3.30
CLEC	2.17	2.98
Difference	-0.53	

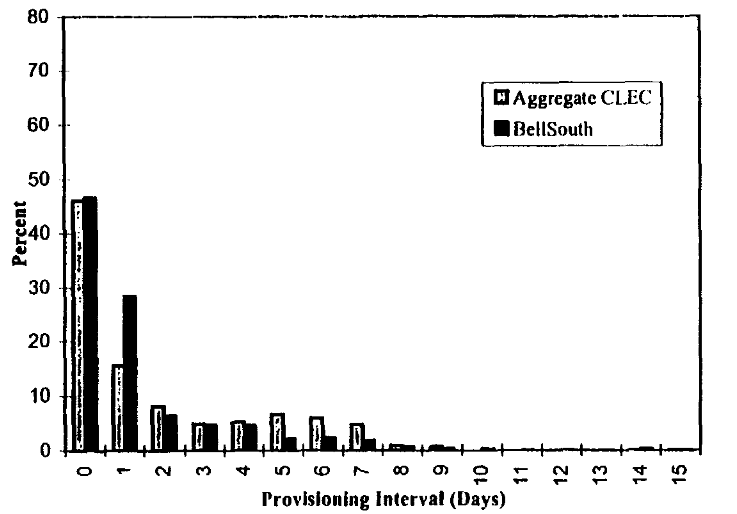
Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	-11.54	0.0000
FCC	-11.57	0.0000
BST	-6.59	0.0000

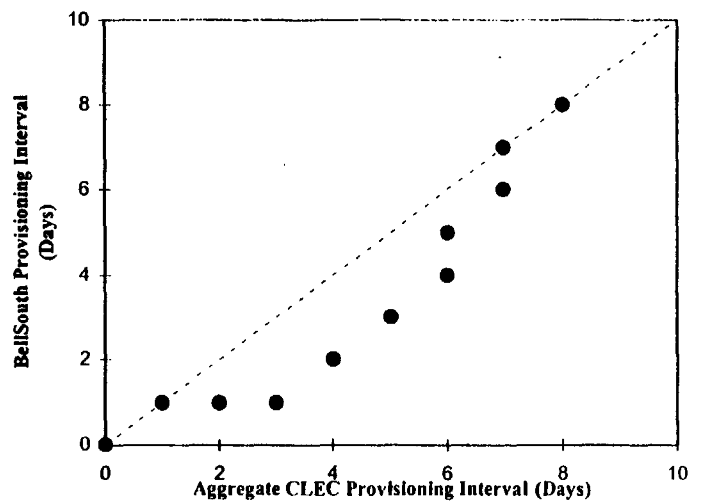
Data used in analysis does not include any records with missed appointments due to customer rescheduling or records corresponding to official services.
The application of statistical trimming removed records with completion interval-provisioning of above 99 days. This resulted in the removal of no CLEC records and 0.004% of the BellSouth records.

Adjusted
September BellSouth and CLEC Completion Interval-Provisioning
Baton Rouge Cases

Frequency Distribution



Quantile Comparison



Descriptive Measures

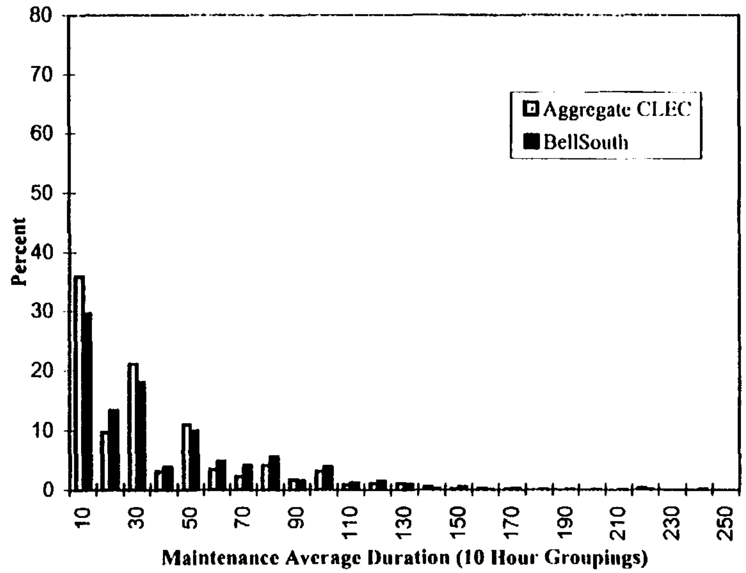
Service Provider	Mean	Standard Deviation
BST	1.45	2.62
CLEC	1.95	2.64
Difference	-0.50	

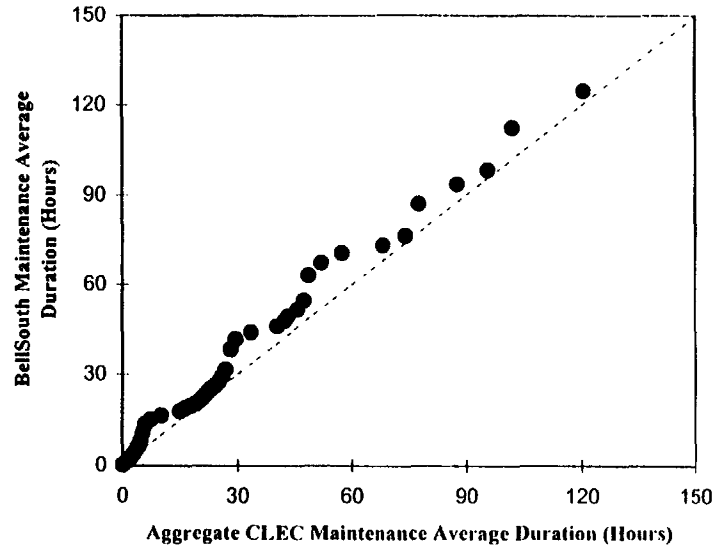
Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	-10.08	0.0000
FCC	-10.07	0.0000
BST	-3.15	0.2350

Data used in analysis does not include any records with missed appointments due to customer rescheduling or records corresponding to official services.
The application of statistical trimming removed records with completion interval-provisioning of above 99 days. This resulted in the removal of no CLEC records and 0.004% of the BellSouth records.

Adjusted
September BellSouth and CLEC Average Duration-Maintenance
Shreveport





Descriptive Measures

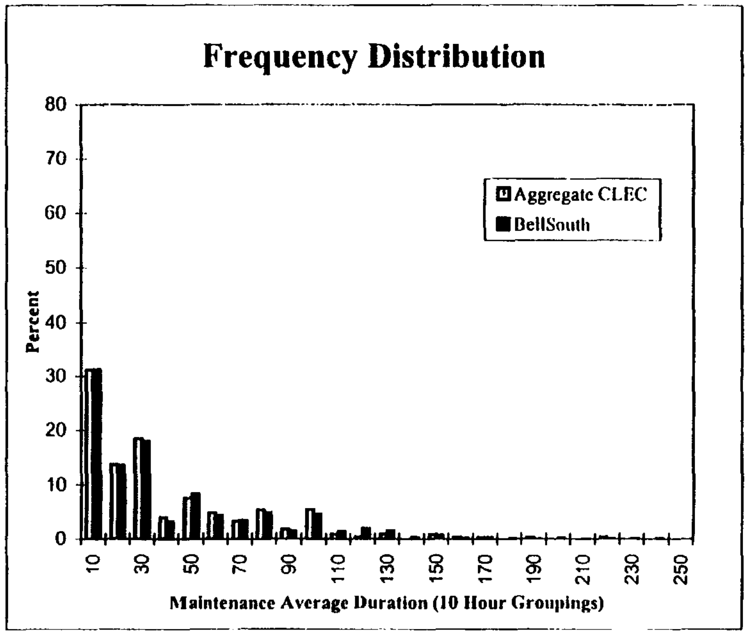
Service Provider	Mean	Standard Deviation
BST	34.71	35.04
CLEC	30.59	33.54
Difference	4.12	

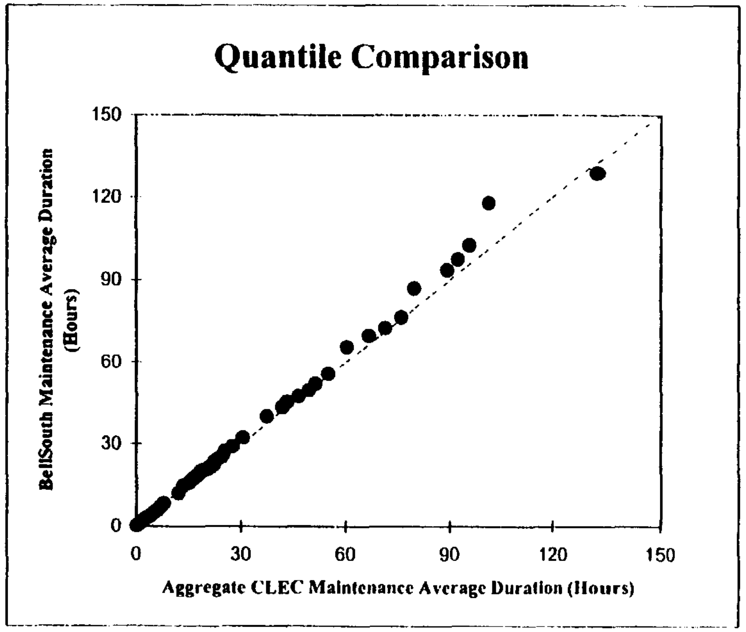
Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	2.79	0.2631
FCC	2.79	0.2606
BST	2.35	1.2757

Data used in analysis includes only direct customer reports. The results exclude in public service lines and durations > 240 hours

Adjusted
September BellSouth and CLEC Average Duration-Maintenance
Lafayette





Descriptive Measures

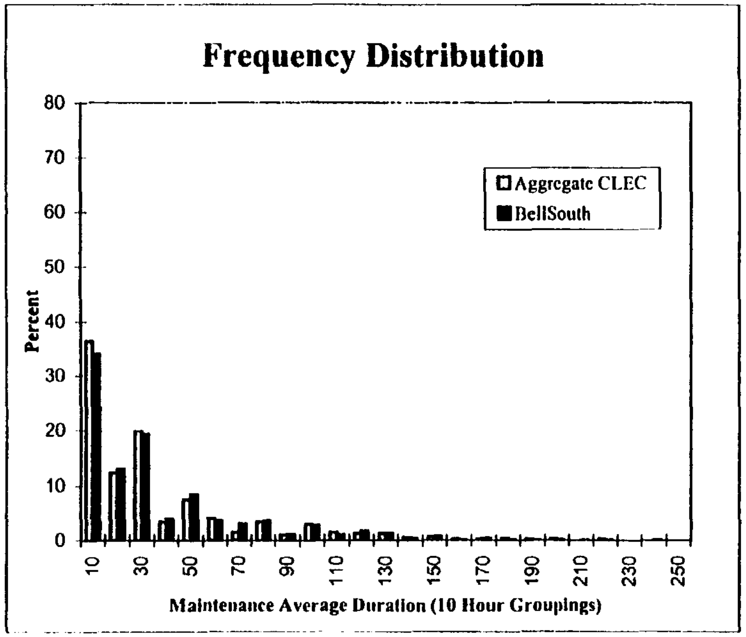
Service Provider	Mean	Standard Deviation
BST	35.14	36.93
CLEC	34.08	35.99
Difference	1.07	

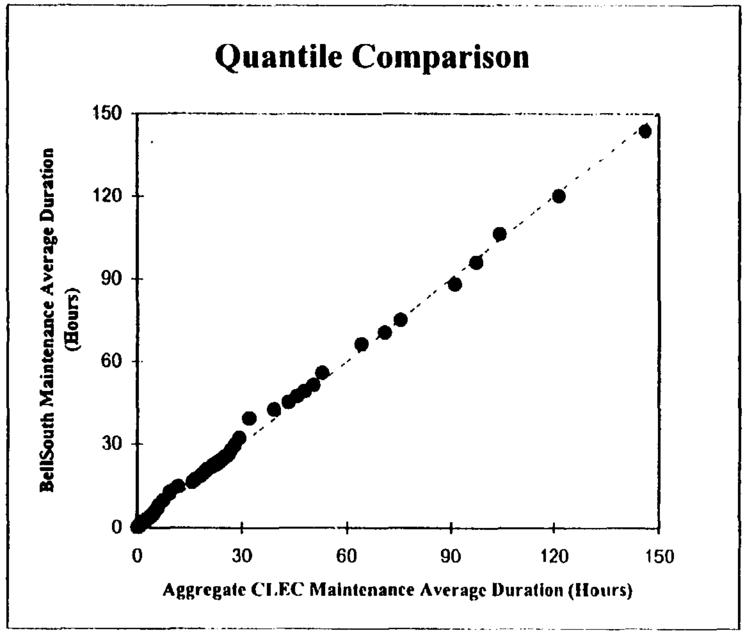
Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	0.52	30.1862
FCC	0.52	30.1759
BST	0.40	34.6836

Data used in analysis includes only direct customer reports. The results exclude in public service lines and durations > 240 hours

Adjusted
September BellSouth and CLEC Average Duration-Maintenance
New Orleans





Descriptive Measures

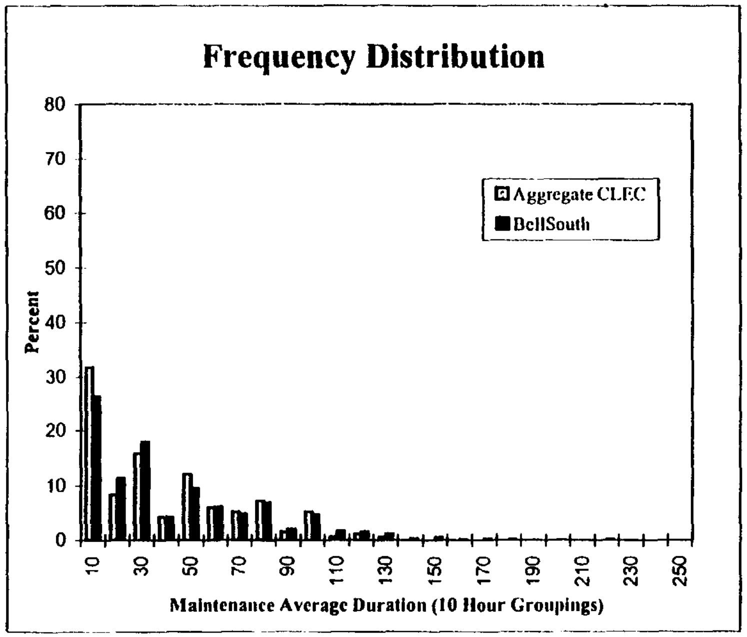
Service Provider	Mean	Standard Deviation
BST	32.59	37.19
CLEC	32.12	38.20
Difference	0.47	

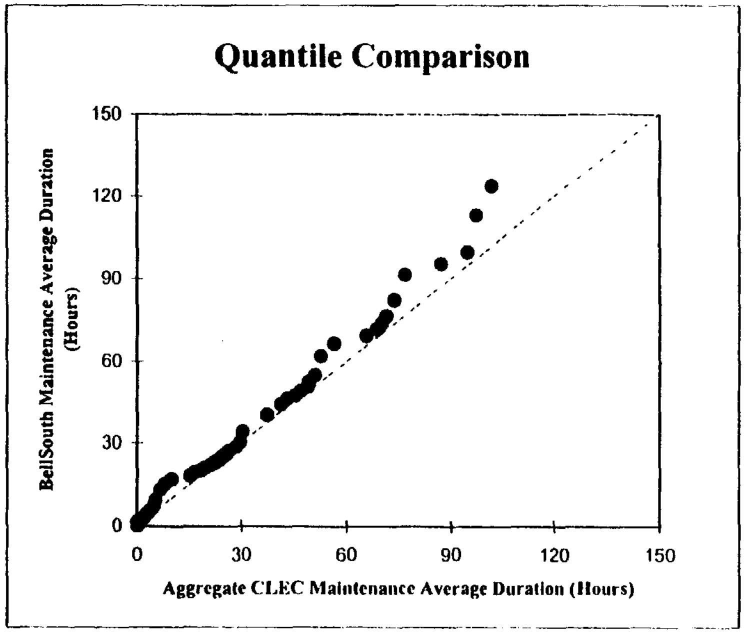
Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	0.33	37.0821
FCC	0.33	37.0881
BST	0.21	41.7217

Data used in analysis includes only direct customer reports. The results exclude in public service lines and durations > 240 hours

Adjusted
September BellSouth and CLEC Average Duration-Maintenance
Baton Rouge





Descriptive Measures

Service Provider	Mean	Standard Deviation
BST	38.06	35.01
CLEC	34.16	30.96
Difference	3.90	

Analytic Measures

Testing Method	Test Statistic	P-value (percent)
LCUG	2.05	2.0178
FCC	2.06	1.9922
BST	1.34	9.7173

Data used in analysis includes only direct customer reports. The results exclude in public service lines and durations > 240 hours

Appendix J
Aggregate Assessment of Nondiscrimination - Multiple Testing Isssues

that the resulting overall false alarm rate is no higher than the desired level,

4. show that other problems are encountered when the alternative method is used with too many tests, and
5. recommend that the total number of tests used to judge nondiscrimination be kept to a small number of independent tests, perhaps one from each of the main service quality measurement categories.

Lack of independence

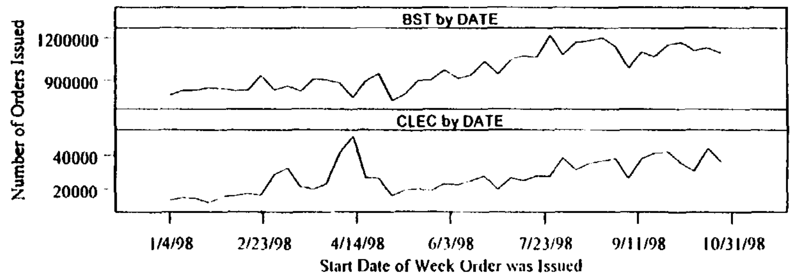
Many performance measures within the same Service Quality Measurement categories are calculated from a common set of data. While the measures quantify different aspects of performance, the fact that certain common variables are used in the calculations suggests that the measures will be correlated.

The Order Completion Interval, the Held Order Interval, and the Jeopardy Notice Interval all get quantified in two ways: by the average value, and by the distribution of the number of days in the interval. If, for example, parity tests of both the average and the proportion of intervals greater than five days are both included in an aggregation of tests, then there would be dependencies at least between the measurement pairs for each type of interval.

The Percent Missed Installation Appointments and the Order Completion Interval are also confounded. Those orders that have missed installation appointments will have longer completion intervals.

As for the independence of a particular measure between consecutive months, one needs to consider business trends over time. Figure 1 shows the number of weekly BST and CLEC service requests for the whole BellSouth region over the first ten months of 1998.

Figure 1 - Number of Weekly Service Request During the First Ten Months of 1998



It is apparent that both the BST and CLEC series exhibit both an increasing trend, as well as some oscillations about that trend. To get a clearer picture of this, we can decompose each series into a trend, oscillatory, and remainder components.

We can do this by using repeated loess fitting as described by Cleveland.[3] Figure 2 show the results of this decomposition for the BellSouth series. Figure 3 show the CLEC results.

[3] Cleveland, W. S. (1993), *Visualizing Data*. Hobart Press, Summit, New Jersey.

1. the number of allowed individual parity test failures in a month, denoted by k_1,
2. the number of allowed three-consecutive-month failures of a parity test, denoted by k_2, and
3. the common false alarm rate of the individual tests, denoted by α_1.

AT&T suggests that k_2 be set to zero, arguing that the expected number of parity tests that fail in three consecutive months is small. This calculation assumes independence of tests from month-to-month.

The overall false alarm rate, α, is a function of

a) the three values k_1, k_2, α_1, and

b) the total number of individual parity tests, N.

By setting $k_2 = 0$, and assuming independence of tests within a month, as well as independence across consecutive months, the equation can be written as

$$\alpha = 1-(1-\alpha_1^3)^N \cdot P(k_1, N, p).$$

$P(k_1,N,p)$ is the cumulative binomial distribution. This gives the probability that there are at most k_1 false parity test failures out of N total parity tests when the probability of an individual false parity test failure is p. The false parity test failure probability, p, is computed as

$$p = \frac{\alpha_1 - \alpha_1^3}{1-\alpha_1^3}.$$

By using this function, values of k_1 and α_1 can be found that provide a desired value of α.

For example, suppose that N = 100 parity tests are to be performed with an overall false alarm rate of 5 percent. Then it can be shown that $k_1 = 8$, and $\alpha_1 = 0.0460$ (4.6 percent).˙ If an individual parity measure is calculated by standardizing the difference of average BellSouth and average CLEC performance (where the CLEC value is subtracted from the BellSouth value), then a conclusion of discriminatory behavior is reached if the parity measure is "too small."

The notion of "too small" is quantified by finding the value, C, in the parity measure distribution for which 100α percent of all values are less than it.[4] Under the right conditions, the parity measure distribution can be considered to be a standard normal distribution. In the previous example, the false alarm rate was 4.6 percent. Using a standard normal distribution, the critical value for the test is C = -1.685.

To see what happens when dependence exists between a set of parity tests within a given month, we performed a simple simulation experiment. Since we are only simulating parity measures within a month, the equation for determining k_1 and α_1 simplifies to

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[4] This assumes that one wants to have a one tailed test. If a two tailed test is desired, then the point of discrimination is reached at the value of the parity measure distribution for which 100(α/2) percent of all values are less than it.

Table 1 - Summary of Simulation Results, the Consequences of Assuming Independence when Parity Tests are Correlated

Total Number of Tests N	Number of Allowable Test Failures k_1	Individual False Alarm Rate $100\alpha_1\%$	Critical Value C	Estimated Overall False Alarm Rate $100\alpha\%$
5	0	1.02	-2.3187	5.61
10	1	3.68	-1.7894	6.93
50	4	4.02	-1.7479	7.78
100	8	4.78	-1.6670	8.45
500	32	4.87	-1.6577	9.92
1000	61	4.99	-1.6455	9.55

The desired overall false alarm rate is 5 percent.

These results are only good for the type of correlation that was assumed to exist between parity measures. The correlation structure that is described above was chosen because it has a uniform mix of correlation levels between the parity measures.

While there is evidence that correlation exists between some parity measures, we do not know the exact nature of the structure across a set of parity measures. Thus, this simulation is only an example of what can happen to the overall false alarm rate when procedures based on independence of parity measures are used.

Alternative Procedures

If the distribution of the N monthly parity measures are reasonably approximated by a multivariate normal distribution, then one can use Scheffé's S-Method of multiple comparisons.[5] This method depends upon inverting a correlation matrix. If one wants to have a computational feasible problem, then a small number of parity tests should be considered.

If there is concern about the appropriateness of using the multivariate normal distribution to model the distribution of the N monthly parity measures, then one can employ the Bonferroni inequality.[6] This is a relationship which holds whether or not the individual parity tests are independent.

Let $Z_1, \ldots, Z_N$ be the results of N monthly parity measures, C be the common critical value for the parity tests, and α_1 the common false alarm rate for each parity test. If one sided tests are being performed, the Bonferroni inequality can be written as

$$1 - P(Z_1 \geq C, \ldots, Z_N \geq C) \leq \sum_{i=1}^{N} P(Z_i < C) = N \cdot \alpha_1 .$$

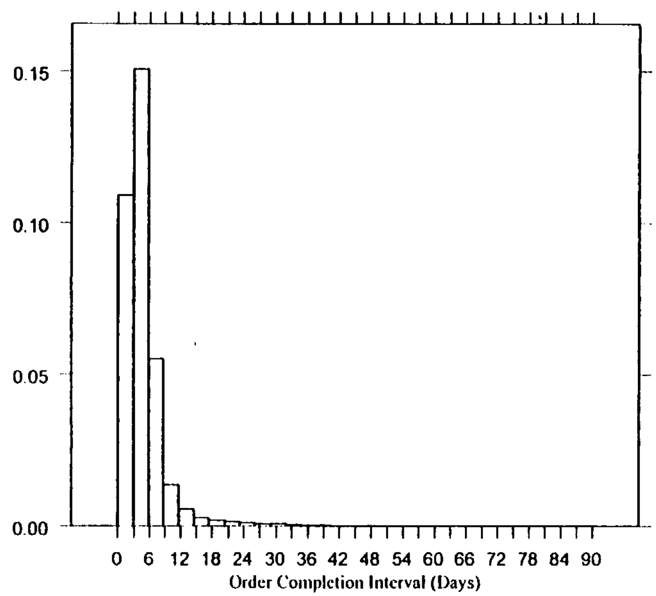
The left side of this relationship is the probability of having at least one parity tests out of N fail. The relationship implies that if you do not allow any parity test failures out of the N monthly tests, then the overall false alarm rate when performing multiple comparisons is no more than

$$\alpha = N \cdot \alpha_1 .$$

[5] Scheffé, H. (1959), *The Analysis of Variance*, J. Wiley & Sons, Inc., New York.

[6] The Bonferroni inequality is discussed in numerous probability and statistics text books. For example, Mendenhall, W., Scheaffer, R.L., and Wackerly, D. D. (1986), *Mathematical Statistics with Applications, Third Edition*, Duxbury Press, Boston.

Figure 4 - Distribution of BellSouth's Order Completion Interval for Dispatched, Residential Orders with Less Than 10 Circuits



The simulation was conducted using the following steps.

1. Draw a sample of size 8,000 from the OCI distribution. This represents the BellSouth orders for the month.
2. Compute $\bar{x}_B$ and s_B, the sample mean and standard deviation of the BellSouth sample.
3. Draw a sample of 500 from the OCI distribution. This represents the CLEC orders for the month.
4. Compute $\bar{x}_C$, the sample mean of the CLEC sample.
5. Compute the LCUG parity measure

$$z = \frac{\bar{x}_B - \bar{x}_C}{s_B\sqrt{\frac{1}{8000} + \frac{1}{500}}}.$$

6. Repeat steps (1) through (5) 100,000 times, storing the z scores.

Figure 2 is a Normal Q-Q Plot of the 100,000 z scores. This is a plot of the estimated quantiles of the parity measure distribution against the same quantiles of the standard normal distribution. If the distribution of the parity measure is normal, the plot should look like a straight line.

The plot shows that the parity measure distribution differs from a normal distribution in the extreme tails. This, though, is the region that determines the critical value for individual tests if the Bonferroni method is used with a large number of tests.

Conclusions

The quantification of performance is an important aspect of quality management. Therefore it is important that BellSouth continue to measure its performance in many different ways.

When it comes to making judgements as to whether or not BellSouth is meeting its nondiscriminatory obligation with respect to the service it provides CLECs and their customers, there are potential problems that can arise when the results of too many parity tests are aggregated. These problems include: dependencies that exist between parity tests, dependencies between consecutive monthly measurements, and parity measures with non-normal distribution.

Our analysis indicates that these problems are negligible when the results of only five to ten parity tests are aggregated in any given month. Furthermore, to guard against dependencies between parity test, a methodology based on the Bonferroni inequality should be used in the aggregation process.

It is useful to point out that both the Bonferroni methodology and the AT&T proposed methodology are approximately the same when only five parity tests are aggregated. When applying AT&T's procedure to five parity tests, no failures are allowed within a month, and the false alarm rate for each individual test is 1.02 percent. A Bonferroni approach would call for pretty much the same procedure – the individual false alarm rate, though, is exactly 1 percent.

Also, if the number of tests is under ten, then the individual test false alarm rate will be greater than 0.5 percent when a Bonferroni procedure is used. This means that the critical value for the individual tests will not come from the extreme tail of a theoretical distribution like the standard normal or Student's t distribution. This is important since simulations suggest that the distribution of extreme values for some parity scores are not modeled well by these distributions.

With respect to comparing parity tests over time, more information is need before we can recommend a procedure. For example, data from more months should be examined to determine the extent of dependencies between monthly parity test results.

Appendix K
Glossary of Acronyms and Statistical Terms

Critical Value: The value of the test statistic that separates the acceptance region from the rejection region.

Critical Region: A region of test statistic values for which the null hypothesis is rejected. Also called the rejection region.

Degrees of Freedom: Relates to the calculation of the **variance** -- (n - 1) deviations from the mean.

Estimate: An estimate is any value calculated from a sample.

Favor: **Statistically Significant** differences that are +2 or larger are defined to be differences which "favor" the CLECs; those that are -2 or smaller are defined to be differences which "favor" BellSouth.

(Relative) Frequency Distribution: An initial indication of what the data look like, that is how the data are distributed. A frequency distribution indicates the number of observations falling within a given class. A **relative frequency distribution** shows the proportion of observations that fall into each class.

Heavy Tailed Distribution: See **normal distribution.** A concentration of observations at one end of the distribution. For example, a distribution of the weights of elephants at a zoo would probably have mostly large weight values and few small values. The distribution of this data would have a heavy tail on the right side, indicating a disproportionate number of observations with large values.

Homoscedasticity: If all the error terms have the same variance, the errors are homoscedastic. If the error terms do not have the same variance, they are called **heteroscedastic**.

Independence / Dependence: Observations A & B are said to be independent when the value of observation A has no influence on the value of observation B. Observations C & D would be dependent if the value of observation C influences the value of observation D, or vise versa.

Least Trimmed Squares Regression[1]: A regression technique introduced in Rousseeuw (1984). This regression method minimizes the sum of the q smallest squared residuals, where q is an integer between (roughly) n/2 and n. This method is robust in that it guards against extreme outliers influencing the functional fit.

Mean: The average value of a set of quantitative data.

Normal Distribution: A set of data has a normal distribution if a graph of the distribution produces a bell-shaped curve. Most of the observations are concentrated near the middle (mean) of the distribution and as you move outward from the middle, either left or right, there is gradually less and less data. A **Standard Normal** has a mean of 0 and a variance of 1.

Null Hypothesis: A statistical hypothesis is a statement about one ore more parameters of a population distribution that requires verification. The null hypothesis is the one whose tenability is actually tested.

One- and Two-tailed tests: A statistical test for which the critical region is in either the upper or lower tail of the sampling distribution is called a one-tailed test. If the critical region is in both the upper and lower tails of the sampling distribution, the statistical test is called a two-tailed test.

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[1] Rousseeuw, P.J (1984). Least median of squares regression. Journal of the American Statistical Association, 79, 871-881.

set of data by examining how the data change over time and if there is a describable pattern of behavior over time.

Variance: A summary statistic for measuring variation in a set of data. This measure of central tendency measures the average of the square deviations from the mean. See **standard deviation**.